Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 5, Issue 3; January-March, 2018, pp. 245-250

http://www.krishisanskriti.org/Publication.html

Mathematical Modelling of a Fuzzy Logic Based Smart Structure to Minimize its Vibration

Rohit Agrawal[1] and Sankha Bhaduri[2]

[1,2]*Birla Institute of Technology, Mesra Birla Institute of Technology, Mesra*
E-mail: [1]rohitagrawal929@gmail.com, [2]sbhaduri@bitmesra.ac.in

Abstract—*In this study of an active vibration control systemis modeled to minimize the amplitude of vibrationof structure. Piezoelectric Lead Zirconate Titanate(PZT) patch is used as an actuator to control theamplitude of vibration. The PZT patch ispasted at the top surface of a cantilever beam.The displacement at the tip end of the cantileverbeam is measured using a non -contact inductivedisplacement sensor. During the upward motion ofthe cantilever beam the voltage is supplied to the*
PZT patch. Due to the voltage supplied to the PZTpatch the length of the patch will be increasedand it will give a bending moment on the cantileverbeam. The direction of the moment applied on thebeam is such that it opposes the displacementof the beam. Depending on the velocity anddisplacement at the free end of the beam,the fuzzy logic control technique is used tocontrol the voltage to the actuator. It isobserved that the proposed model successfullyminimizes the amplitude of vibration of thebeam under the action of different types of forces.

Keywords*: Smart Beam, Active Vibration control, Piezoelectric Patch, Fuzzy Logic Control.*

1. INTRODUCTION

Active vibration control is a process in which amplitude of vibration of a structure is minimized by applying actuating force to the structure that is appropriately out of phase but equal in amplitude to the original force. The two equal and opposite forces cancelled and structure stops vibrating.

S. Belouettaret al. [1]focused on active controlof sandwich piezoelectric–elastic-piezoelectric beams of the linear and nonlinear vibrations. Aydin Azizi et al. [2] used piezoelectric material as sensors and actuator to minimize the vibration ofbeam. They used different types of controller on vibration.Zhi-cheng Qiuet al. [3]studied the active vibration suppression of a beam bonded withpiezoelectric patch as sensor/actuators.

To minimize the vibration of a composite plate with the help of piezoelectric actuator Tran Ich Thinh et al. [4]developed a FEMbased on the shear deformation theory.Li Sui and Xin Xiong Gengchen Shi [5] studied piezoelectric actuator is a device dependingupon counter piezoelectric effect, when voltage is applied, it will expand.Saurabh Kumar et al. [6] deals with the active vibration control of a cantilever beam bonded with two piezoelectric patches. They found that the best result is obtained when piezoelectric patches are bonded near the fixed end of the cantilever beam. The Hamilton's principle and Galerkin's method was used byM. Kerboua et al. [7]for the composite cantilever beam with the help of piezoelectric patch.A new methodology was proposed by Osama Abdeljaber et al. [8] for flexible cantilever plates. In this research work it has been discussed about the active control system of cantilever beam with the help of piezoelectric actuator/sensors for implementing new technologies for minimizing vibration.

In this study Piezoelectric Lead Zirconate Titanate (PZT) patch materials is used as an actuator to minimize the amplitude of vibration of a beam. The piezoelectric patch is pasted at the top surface of the cantilever beam. A piezoelectric material expands when voltage is applied to it. Then, it will give bending moment on the cantilever beam. The direction of the bending moment applied on the beam is such that it opposes the displacement of the beam. Depending on the velocity and displacement, fuzzy logic control technique is used here to control the voltage to the actuator.

2. ANALYSIS

Fig. 1 shows the schematic diagram of the arrangements made to minimize the amplitude of vibration of a cantilever beam. The velocity and displacement of the free end of the cantilever beam are measured using sensors and are passed to the fuzzy logic controller. Depending on the displacement and velocity at the free end of the beam fuzzy logic controller will send voltage to the piezoelectric patch. During the upward movement of the beam positive voltage is applied on the PZT patch. Therefore, the length of the patch will be increased and it will apply a bending moment just opposite to the direction of the movement of beam. Using this principle, the amplitude of vibration of structure is minimized.

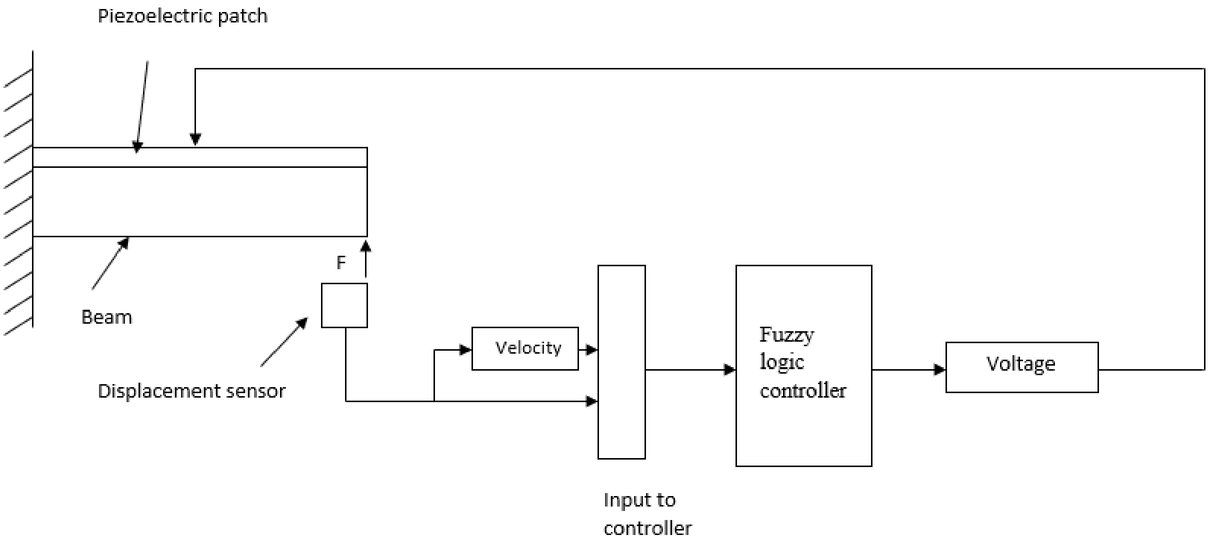


Fig. 1: Schematic diagram shows arrangement of beam along with actuator and controller

2.1 SDOF model:

$m\ddot{x} + c\dot{x} + kx = F_{dis} - F_{actu}$

$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}[F_d - F_{actu}]$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}\ddot{x}\\\dot{x}\end{bmatrix} + \begin{bmatrix}\frac{c}{m} & \frac{k}{m}\\-1 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\x\end{bmatrix} = \begin{bmatrix}\frac{1}{m}\\0\end{bmatrix}[F_d - F_{actu}]$$

$$\begin{bmatrix}\ddot{x}\\\dot{x}\end{bmatrix} = -\begin{bmatrix}\frac{c}{m} & \frac{k}{m}\\-1 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\x\end{bmatrix} + \begin{bmatrix}\frac{1}{m}\\0\end{bmatrix}[F_d - F_{actu}]$$

In state space representation of vibration,

$[\dot{X}] = [A]\{X\} + [B]\{U\}$

where, $A = -\begin{bmatrix}\frac{c}{m} & \frac{k}{m}\\-1 & 0\end{bmatrix}$, $B = \begin{bmatrix}\frac{1}{m}\\0\end{bmatrix}$

$[Y] = [C]\{X\} + [D]\{U\}$

$C = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ and $D = \begin{bmatrix}0\\0\end{bmatrix}$

2.2 MDOF model:

$[M]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{F_{dis}\} - \{F_{actu}\}$

$\ddot{x} + \frac{c}{[M]}\dot{x} + \frac{k}{[M]}x = \frac{1}{[M]}[F_d - F_{actu}]$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}\ddot{x}\\\dot{x}\end{bmatrix} + \begin{bmatrix}\frac{c}{[M]} & \frac{k}{[M]}\\-1 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\x\end{bmatrix} = \begin{bmatrix}\frac{1}{[M]}\\0\end{bmatrix}[F_d - F_{actu}]$$

$$\begin{bmatrix}\ddot{x}\\\dot{x}\end{bmatrix}_{16X1} = -\begin{bmatrix}[M]^{-1}[c] & [M]^{-1}[k]\\-1 & 0\end{bmatrix}_{16X16}\begin{bmatrix}\dot{x}\\x\end{bmatrix}_{16X1} + \begin{bmatrix}[M]^{-1}\\ [0]\end{bmatrix}_{16X1}[F_d - F_{actu}]$$

In state space representation of vibration,

$[\dot{X}] = [A]\{X\} + [B]\{U\}$

Here, $A = -\begin{bmatrix}[M]^{-1}[c] & [M]^{-1}[k]\\-1 & 0\end{bmatrix}_{16X16}$

$B = \begin{bmatrix}[M]^{-1}\\ [0]\end{bmatrix}_{16X1}$

MDOF system is modeled by beam element.

The stiffness and mass matrices of piezoelectric beam element are:

$$[k_p] = \frac{E_p I_p}{{l_p}^3}\begin{bmatrix}12 & 6l_p & -12 & 6l_p\\6l_p & 4{l_p}^2 & -6l_p & 2{l_p}^2\\-12 & -6l_p & 12 & -6l_p\\6l_p & 2{l_p}^2 & -6l_p & 4{l_p}^2\end{bmatrix}$$

$$[M_p] = \frac{\rho_p A_p l_p}{420}\begin{bmatrix}156 & 22l_p & 54 & -13l_p\\22l_p & 4{l_p}^2 & 13l_p & -3{l_p}^2\\54 & 13l_p & 156 & -22l_p\\-13l_p & -3{l_p}^2 & -22l_p & 4{l_p}^2\end{bmatrix}$$

The smart beam is obtained by pasting the piezoelectric patch on top of the regular beam as shown in Fig.:2

EI= $E_b I_b + E_p I_p$ is Flexural rigidity

ρA =$b_b(\rho_b t_b + \rho_p t_p)$

t_p= thickness ofpatch.

The stiffness and mass matrices of smart beam element are:

$$[k] = \frac{EI}{{l_b}^3}\begin{bmatrix} 12 & 6l_b & -12 & 6l_b \\ 6l_b & 4{l_b}^2 & -6l_b & 2{l_b}^2 \\ -12 & -6l_b & 12 & -6l_b \\ 6l_b & 2{l_b}^2 & -6l_b & 4{l_b}^2 \end{bmatrix}$$

$$[M]= \frac{\rho A l_b}{420}\begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4{l_b}^2 & 13l_b & -3{l_b}^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3{l_b}^2 & -22l_b & 4{l_b}^2 \end{bmatrix}$$

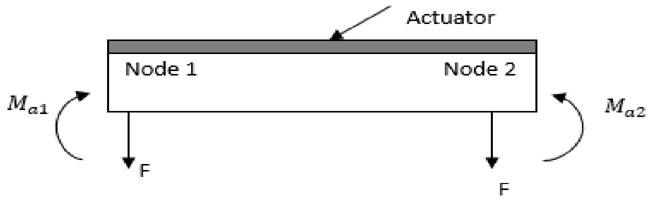


Fig.:2

3. FUZZY LOGIC CONTROLLER

Fuzzy controller block diagram is shown in Fig.3

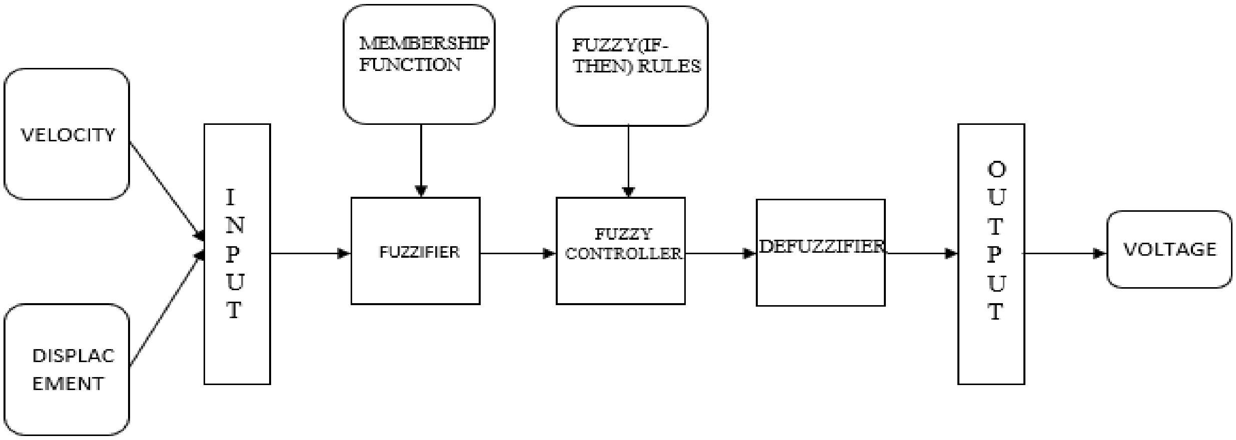


Fig. 3: Fuzzy controller block diagram

Velocity and displacement are input to the system and voltage is the output from the system. Triangular membership is used in this controller. Centroid method is used for fuzzification.

3.1 RULES OF FUZZY CONTROLLER

Velocity and displacement are input and voltage is output. Fuzzy controller executes the rule and according to rule base gives output. The purpose of fuzzy controller is to deliver the voltage to the actuator to reduce the amplitude of vibration.

There are several rules obtained as follow:

1) If Velocity is PH and Displacement is PS, the beam is above the equilibrium position. then supply the Voltage to the actuator.
2) If Velocity is PM and Displacement is PM, the beam is above the equilibrium position .so it needs to supply the Voltage to the actuator.
3) If Velocity is PS and Displacement is PH, the beam is above the equilibrium position .so it needs to supply the Voltage to the actuator.
4) If Velocity is negative and Displacement is PH, then supply the Voltage ZE to the actuator.
5) If Velocity is negative and Displacement is PM, then supply the Voltage ZE to the actuator.
6) If Velocity is negative and Displacement is PS, then supply the Voltage ZE to the actuator.
7) If Velocity is negative and Displacement is negative, then supply the Voltage ZE to the actuator.
8) If Velocity is PS and Displacement is negative, then supply the Voltage ZE to the actuator.
9) If Velocity is PM and Displacement is negative, then supply the Voltage ZE to the actuator.
10) If Velocity is PH and Displacement is negative, then supply the Voltage ZE to the actuator.
11) If Velocity is ZE and Displacement is PH, the beam is above the equilibrium position. so, supply the Voltage to the actuator.

4. ACTUATOR EQUATION

When only a single patch is used, the beam is subject to both bending moment and an axial force shown in [9].

Strain distribution,$\varepsilon(z)$= αz + ε_0,where first part is due to bending and ε_0 is longitudinal strain shown in Fig.:4

The stress distribution in the beam and the piezo-patch are:

$$\sigma_b(\mathrm{z}) = E_b(\alpha z + \varepsilon_0)$$

and $$\sigma_p(\mathrm{z}) = E_p(\alpha z + \varepsilon_0 - \varepsilon_p)$$

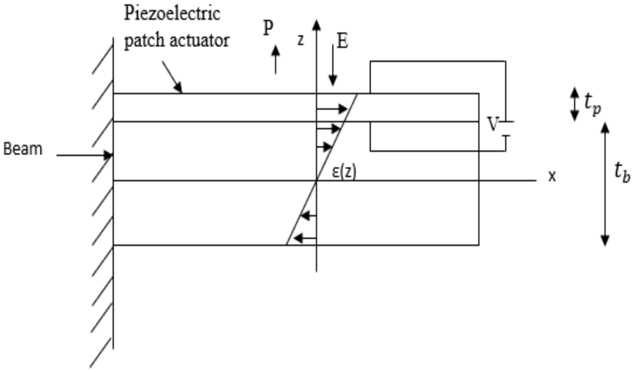


Fig.:4

The moment and force equations are written as:

$$\int_{-\frac{t_b}{2}}^{\frac{t_b}{2}} \sigma_b\,(z) w_b \mathrm{zdz} + \int_{\frac{t_b}{2}}^{\frac{t_b}{2+t_p}} \sigma_p\,(z) w_p \mathrm{zdz} = 0$$

$$\int_{-\frac{t_b}{2}}^{\frac{t_b}{2}} \sigma_b\,(z) w_b \mathrm{zdz} + \int_{\frac{t_b}{2}}^{\frac{t_b}{2+t_p}} \sigma_p\,(z) w_p \mathrm{zdz} = 0$$

Finally, the moment distribution induced in the section of beam under the piezo-patch is written as:

$$M_p = E_b \mathrm{I}\alpha = K_a \mathrm{V}$$

Here,

$$K_a = \left[\frac{6 w_b t_b^2 E_b (1+h)}{4+6h+4h^2+k_e+\frac{1}{k_b}}\right]\left(\frac{d_{31}}{t_p}\right)$$

$$K_a = \mathit{the\ actuator\ constant}$$

where $k_b = \left(\frac{\frac{1}{12} E_b\, w_b\, t_b^3}{\frac{1}{12} E_p\, w_p\, t_p^3}\right)$ is ratio of flexural rigidity of beam and the patch.

where $k_e = \left(\frac{E_b w_b t_b}{E_p w_p t_p}\right)$ is ratio of extensional rigidity of beam and patch.

h=$\frac{t_p}{t_b}$ is thickness ratio.

5. MATLAB SIMULINK MODEL

Table 1: Properties of steel cantilever beam and piezoelectric patchesThe performance analysis of the single piezoelectric actuator is done using matlab simulation. Thefollowing Fig.5 shows the block diagram for the Simulink model.

Parameters	Cantilever steel beam	Piezo-electric actuator
Length	l_b=100mm	l_p=100 mm
Width	b_b=18 mm	b_p=18 mm
Thickness	t_b=15 mm	t_p=3 mm
Young's Modulus of Elasticity	$E_b = 2\times10^5$ N/mm^2	$E_p = 6.66\times10^4$ N/mm^2
Density	ρ = 7.81×10^{-6} kg/mm^3	ρ = 7.4×10^{-6} kg/mm^3
Piezoelectric(PZT-5H) Strain Constant (d31)		d_{31}=265×10^{-9} mm/V
Damping Constants	α=4.76 and β=2.188×10^{-6}	

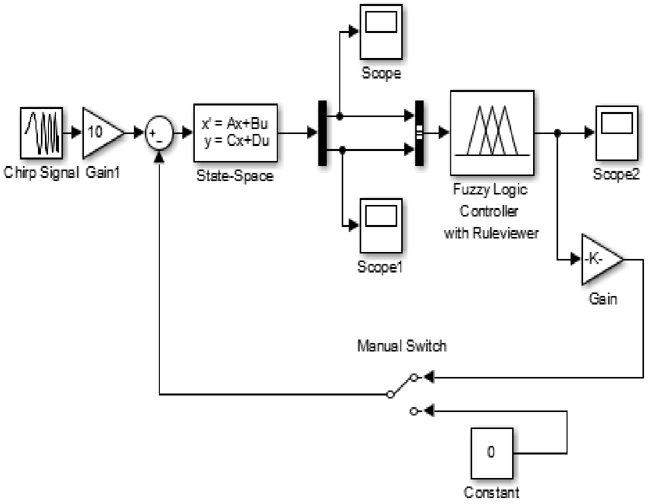
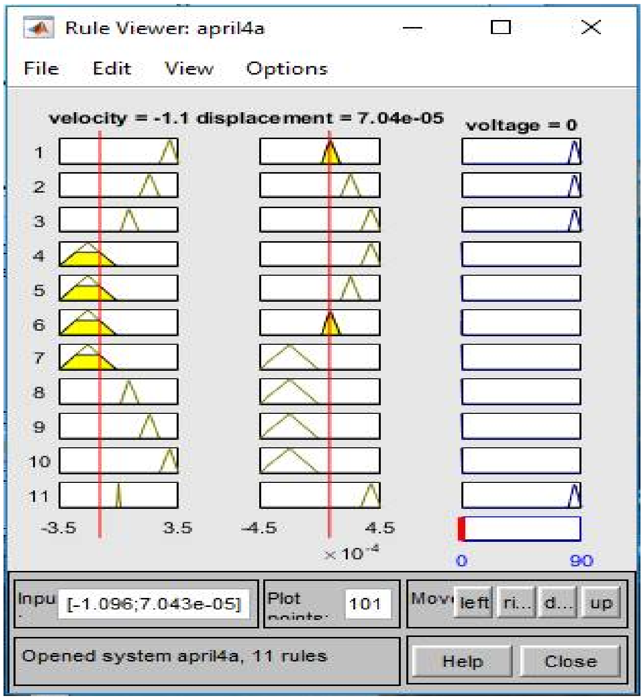


Fig.5: Matlab simulink model.

6. NUMERICAL STUDY

A beam is divided into four finite elements. Here, we have to consider a steel cantilever beam. In SIMULINK, a state space block is used to represent the beam model in state space form. The beam is 100mm in length and 18mm ×15mm in cross-section. The piezoelectric patch is pasted at the top surface of a cantilever beam. The displacement at the tip end of the cantilever beam is measured using a non -contact inductive displacement sensor. During the upward motion of the cantilever beam the voltage is suppliedto the piezoelectric patch. Due to the voltage supplied to the piezoelectric patch the length of the patch will be increased and it will give a bending moment on the cantilever beam. The direction of the moment applied on the beam is such that it opposes the displacement of the beam. Depending on the velocity and displacement at the free end of the beam, the fuzzy logic control technique is used to control the voltage to the actuator. Table-1: shows properties of steel beam and patches.Fig.6 shows rule viewer, Fig.7 shows surface viewer. Fig 8 and Fig.9 showsuncontrolled and controlled response of the chirp signal and step signal respectively.


Rule Viewer: april4a

velocity = -1.1 displacement = 7.04e-05

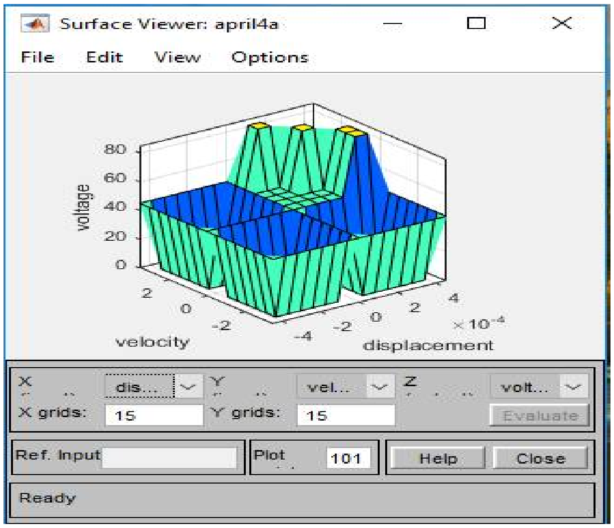
voltage = 0

Input: [-1.096;7.043e-05]

Plot points: 101

Opened system april4a, 11 rules

Fig. 6: Rule viewer


Surface Viewer: april4a

X (input): dis...

Y (input): vel...

Z (output): volt...

X grids: 15

Y grids: 15

Ref. Input:

Plot points: 101

Ready

Fig. 7: Surface viewer

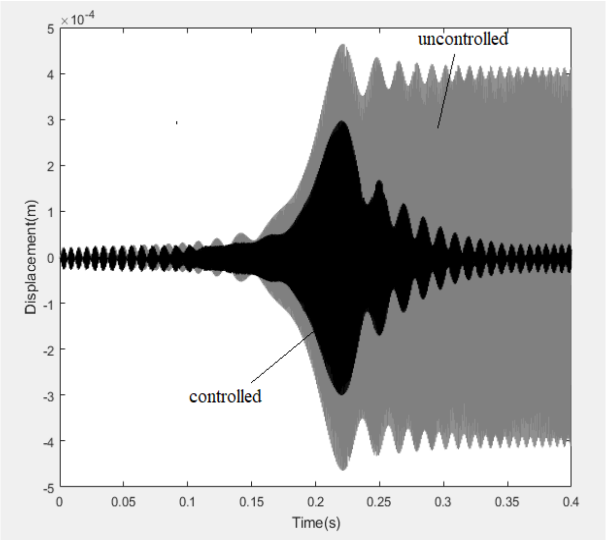


Fig. 8: Uncontrolled and controlled responseunder action of chirp signal.

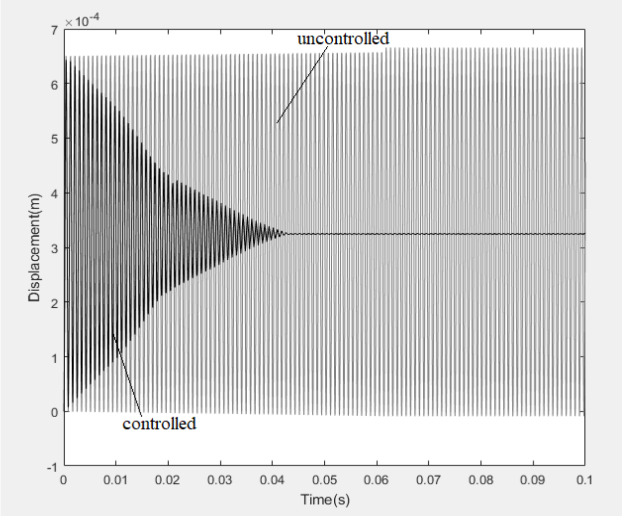


Fig. 9: Uncontrolled and controlled responseunder the action of stepsignal.

7. CONCLUSION

The amplitude of vibration of a cantilever beam is successfully minimized by the proposed piezoelectric actuator for chirp and step signal input. Fuzzy logic controller provides an effective means to control the voltage through the piezoelectric actuator. This work is useful in controlling the vibration of Machine Tools, Plane wings, engineering structure etc.

REFERENCES

[1] S. Belouettar, L. Azrar, E.M. Daya, V. Laptev, M. Potier-Ferry, "Active control of nonlinear vibration of sandwich piezoelectric beams: A simplified approach", Computers and Structures 86 (2008) 386–397.

[2] Aydin Azizi, Laaleh Durali, Farid Parvari Rad, Shahin Zareie, “Control of vibration suppression of a smart beam by piezoelectric elements”, 2009 Second International Conference on Environmental and Computer Science, IEEE Computer Society

[3] Zhi-cheng Qiu, Jian-da Han , Xian-min Zhang , Yue-chao Wang , Zhen-wei Wu , “Active vibration control of a flexible beam using a non-collocated acceleration sensor and piezoelectric patch actuator”, Journal of Sound and Vibration 326 (2009) 438–455

[4] Tran Ich Thinh, Le Kim Ngoc, “Static behavior and vibration control of piezoelectric cantilever composite plates and comparison with experiments”, Computational Materials Science 49 (2010) S276–S280

[5] Li Sui, Xin Xiong Gengchen Shi., “Piezoelectric Actuator Design and Application on Active Vibration Control” , Physics Procedia 25 (2012) 1388 – 1396.

[6] Saurabh Kumar, Rajeev Srivastava, R.K.Srivastava, “Active vibration control of smart piezo cantilever beam using pid controller” , International Journal of Research in Engineering and Technology, Volume: 03 Issue: 01 | Jan-2014, eISSN: 2319-1163 | pISSN: 2321-7308

[7] M. Kerboua, Pr A. Megnounif, Pr M. Benguediab , Vibration Control beam using Piezoelectric-based Smart Materials”, Composite Structures

[8] Osama Abdeljaber , OnurAvci , DanielJ.Inman, “Active vibration control of flexible cantilever plates using piezoelectric materials and artificial neural networks” Journal of Sound and Vibration.

[9] Asok Kumar Mallik and Shyamal Chatterjee, “ Principles of Passive and Active Vibration Control” East-West-Press.